Multidomain Verification of Dynamic Signatures Using Local Stability Analysis

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***Abstract*—This paper presents a new approach for online signature verification that exploits the potential of local stability information in handwritten signatures. Different from previous models, this approach classifies a signature using a multidomain strategy. A signature is first split into different segments based on the stability model of a signer. Then, according to the stability model, for each segment, the most profitable domain of representation for verification purposes is detected. In the verification stage, the authenticity of each segment of the unknown signature is evaluated in the most profitable domain of representation. The authenticity of the unknown signature is then determined by combining local verification decisions. The study was carried out on the signatures in the SUSIG database, and the experimental results, thus, obtained confirm the effectiveness of the proposed approach, when compared with others in the literature.**

***Index Terms*—Dynamic signature verification, dynamic time warping, local stability analysis, stability model.**

I. Introduction

Biometrics refers to the individual recognition based on the inherent physiological or behavioral traits of the person to be recognized [1]. Among biometric traits, handwritten signatures remain the most widespread and well accepted form at a user's disposal. In addition, administrative and financial institutions continue to recognize handwritten signatures as a legal means of verifying an individual's identity [2].

Two categories of signature verification systems can be identified depending on the data acquisition method: Static (offline) and dynamic (online) systems. Static systems perform data acquisition after the signing process has been completed, whereas dynamic systems use online acquisition devices that generate electronic signals, which represent the signature during the writing process. As mobile personal systems integrating online acquisition devices have become commercially available at low cost, online signature verification systems are being sought, as they can support a continuously growing number of applications [3]–[5].

Unfortunately, a handwritten signature is the product of a very complex generation process which depends on the psychophysical state of the signer and the conditions under which the signature apposition process occurs [6], [7]. Hence, the variability when signing one's own name repeatedly is not surprising. Signature vary even though people learn to sign at an early age, and then practice to produce similar signatures [8]. The study of the signature stability can offer a deeper understanding of the human processes underlying the handwriting activity, as well as aid in the design of more effective signature verification systems. For the selection of the best subset of reference signatures among the genuine specimens available [9], [10], for choosing the most effective feature–functions for verification aims [10], and for weighting the verification decision obtained at the stroke level [11]–[13].

Approaches for stability analysis in the literature can be grouped into three categories [14]: feature based, model based, and data based. When feature-based approaches are considered, signature stability is estimated by the analysis of a specific set of characteristics. One feature-based technique for estimating local stability in static signatures first segmented the signature images using an equimass approach. Successively, a multiple-matching strategy was applied in which feature vectors extracted from the corresponding regions of genuine specimens were matched through the cosine similarity [15], [16]. Speeded-up local features were also considered for part-based/local stability analysis, since they can provide useful information about how consistently similar the signatures' local parts are among multiple genuine signatures written by an authentic author [17]. When considering dynamic signatures, a comparative study using a distance-based consistency model on features demonstrated that the pen position, velocity, and inclination have the highest consistency [18]. In addition, other results have demonstrated that position is a stronger characteristic than pressure and pen inclination, when personal entropy is considered [19]. A hidden Markov model (HMM) has been used for computing a model-based stability measure to group and characterize dynamic signatures in classes that can be assigned to signature variability and complexity [20]. This measure has been used to determine whether a signature does or does not contain enough information to be successfully processed by a verification system [21]. Data-based approaches use raw data to perform the analysis of signature stability. When static signatures are considered, the stability of each region of a signature can be estimated by a multiple pattern-matching strategy [22], [23]. The basic idea is to match corresponding regions of genuine signatures in order to estimate the extent to which they are locally different. A preliminary step is used to determine the best alignment of the corresponding regions of signatures in order to diminish any differences among them [23]. Another approach considers that given a genuine signature, any other genuine specimen can be considered as the result of a deformation process that can be analyzed with an optical flow [24]. Therefore, the analysis of the optical flow obtained by matching the genuine signatures with other genuine specimens can provide information about the local

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stability in the signature image, useful for signature verification [25]. When dynamic signatures are considered, the stability regions of signatures can be defined as the longest similar sequences of strokes between a pair of genuine signatures [26]. This definition is based on the assumption that signing is the automated execution of a well-learned motor task and, therefore, repeated executions should ideally produce similar specimens. However, variations in signing conditions can lead to signatures that differ only locally because of short sequences of strokes that exhibit different shapes [27]. Another approach estimates a local stability function of dynamic signatures by using dynamic time warping (DTW) to match a genuine signature with other authentic specimens [28]. In this method, each matching is used to identify what are called direct matching points (DMPs), i.e., unambiguously matched points of the genuine signature. Thus, a DMP can indicate the presence of a small stable region of the signature since no significant distortion is detected locally. Furthermore, the local stability value associated with a point of a signature is determined as the average number of times it is a DMP, when the signature is matched against other genuine signatures [29].

This paper presents a new technique for dynamic signature verification which exploits a multidomain approach, using local stability information: The signature was segmented using the signature stability model. Each segment was verified by considering the domain in which the segment was most stable. The verification of each segment was performed using a decision tree classifier. A majority voting strategy was employed for combining the local verification decisions. Signatures from the SUSIG database were used for the experimental tests.

II. Multidomain Verification Technique

Let

$$S = \{S_1,\ S_2 \ldots S_n \ldots S_n\} \tag{1}$$

be a set of *N* genuine signatures. In this paper, each signature S_n is considered to be sequence of elements

$$S_n = (z_n^1, z_{n,}^2, \ldots z_n^i \ldots z_n^i) \tag{2}$$

where each element z_n^i is a 4-tuple

$$z_n^i = (x_n^i, y_n^i, t_n^i, p_n^i) \tag{3}$$

with

1) x_n^i and y_n^i coordinates of the pen on the writing plane;
2) t_n^i timestamp;
3) p_n^i pressure value.

A. Preprocessing

Preprocessing consisted of two separate stages: value normalization and length normalization. Value normalization was performed for each signature according to the linear normalization algorithm so that each value was reported in the range [0, 1]. Similarly, signature length normalization was performed using the linear interpolation algorithm that made the length of all the signatures equal to *M* (in our case $M = 256$) [30].

B. Feature Extraction

Four function features were extracted in the feature extraction step: displacement (*s*), velocity (*v*), acceleration (*a*), and pressure (*p*). In order to represent a signature in these four domains of representation the following equations were considered:

1) Displacement

$$\begin{aligned} s^i &= \sqrt{(x^{i+1} - x^i)^2 + (y^{i+1} - y^i)^2} \\ i &= 1, 2 \ldots, M - 1 \end{aligned} \tag{4.1}$$

$$s^M = s^{M-1}. \tag{4.2}$$

2) Velocity

$$v^i = \frac{s^i}{(t^{i+1} - t^i)}, \quad i = 1, 2 \ldots, M - 1 \tag{5.1}$$

$$v^M = v^{M-1}. \tag{5.2}$$

3) Acceleration

$$a^i = \frac{v^i}{(t^{i+1} - t^i)}, \quad i = 1, 2, \ldots, M - 1 \tag{6.1}$$

$$a^M = a^{M-1}. \tag{6.2}$$

4) Pressure: No conversion was performed in the pressure domain.

Therefore, the feature extraction step allowed the conversion of the signature representation domains from the space of the 4-tuples *(x, y, t, p)* to the space of the 4-tuples *(s, v, a, p)*:

$$(x, y, t, p) \rightarrow (s, v, a, p). \tag{7}$$

C. Classification

The classification step consisted of two phases. The first phase was the training phase, while the second phase concerned the testing procedure.

1) Training Phase: After feature extraction, each signature S_n of the set (1) was represented by a sequence of elements

$$S_n = (z_n^1, z_{n,}^2, \ldots z_n^i \ldots z_n^I) \tag{8}$$

where each element z_n^i is a 4-tuple

$$(s_n^i, v_n^i, a_n^i, p_n^i) \tag{9}$$

where

s_n^i displacement;
v_n^i velocity;
a_n^i acceleration;
$p^i{}_{\mathrm{n}}$ pressure.

The training phase consisted of two main steps: 1) Prototype selection and 2) stability model construction.

a) Prototype Selection: Two approaches were considered in the literature for prototype selection. When multiple prototypes were considered for each signer, in order to model at best signer variability, prototype selection can be performed by variance evaluation within samples [31]–[33], or correlation analysis among stability functions in the signatures [9]. When a single prototype was considered for each signer, the selection of the optimal prototype can be performed by shape and dynamic feature combination [34], time- and position-based averaging [35], or selecting the genuine specimen with the smallest average difference, when compared with the other true signatures available [36]. In the scope of this study and according to Kim *et al.* [36], we decided to investigate the use of a single prototype as reference for stability model construction. Prototype selection was performed using DTW [37] and for each signature S_n of the set S (see (1)), the following set of distances was computed:

$$\{\mathrm{DTW}(S_n, S_p) | p = 1, 2, \ldots, N; n \neq p\} \tag{10}$$

where $DIST_{n,p} = DTW(S_n, S_p)$ denotes the distance between the signatures S_n and S_p. In order to match the points (s_n^i, v_n^i, a_n^i, p_n^i) and (s_p^j, v_p^j, a_p^j, p_p^j) of S_n and S_p respectively, the euclidean distance was used for DTW, although other measures could be considered based on Mahalanobis distance [38], cosine similarity [39], Levenshtein distance [40], etc. The prototype signature was selected as the signature S_n* for which the average distance with respect to the other specimens was minimum, i.e.,

$$S_{n^*} \rightarrow \mathrm{argmin}_n \frac{\sum_{p=1,..,N; n \neq p} \mathrm{DIST}_{n,p}}{N-1}. \tag{11}$$

b) Stability Model Construction: Now, let S_r and S_t be two genuine signatures. A warping function between S_r and S_t was any sequence of couples of indexes, identifying points of S_r and S_t to be joined [37]:

$$W(S_r, S_t) = c_1, c_2, \ldots, c_K \tag{12}$$

where $c_k = (i_k, j_k)$ (k, i_k, j_k integers, $1 \leq k \leq K$, $1 \leq i_k \leq M$, $1 \leq j_k \leq M$). Now, if we consider a distance measure $d(c_k) = d(z_r^{ik}, z_t^{jk})$ between points of S_r and S_t, we can associate with $W(S_r, S_t)$ the dissimilarity measure

$$D_{W(S_r, S_t)} = \sum_{k=1}^{K} d(c_k). \tag{13}$$

DTW was used to detect the warping function $W*(S_r, S_t) = c*_1, c*_2, \ldots, c*_{K*}$ which satisfied the monotonicity ($i_1 \leq i_2 \leq \ldots \leq i_{K-1} \leq i_K$ and $j_1 \leq j_2 \leq \ldots \leq j_{K-1} \leq j_K$), continuity ($i_k - i_{k-1} \leq 1$, and $j_k - j_{k-1} \leq 1$, for $k = 2, 3, \ldots, K$) and boundary $[c_1 = (1,1), c_K = (M, M)]$ conditions, and for which it was found to be [37]

$$D_{W*(S_r, S_t)} = \min_{W(S_r, S_t)} D_{W(S_r, S_t)}. \tag{14}$$

From $W*(S_r, S_t)$, we identified the DMP of S_r with respect to S_t [29]. A DMP of a signature S_r with respect to S_t was a point which had a one-to-one coupling with a point of S_t. In other words, let z_r^p be a point of S_r coupled with z_t^q of S_t; z_r^p is the DMP of S_r with respect to S_t if

1) $\forall \bar{p} = 1, \ldots, M,, \bar{p} \neq p$, yields:
$z_r^{\bar{p}}$ is not coupled with z_t^q.
2) $\forall \bar{q} = 1, .., M, \bar{q} \neq q$, yields:
$z_t^{\bar{q}}$ is not coupled with $z_r^{\bar{p}}$.

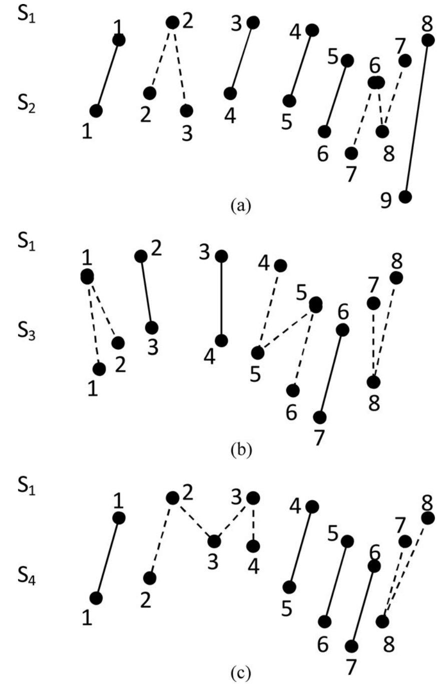

Fig. 1. (a). $W(S_1, S_2)$ (b). $W(S_1, S_3)$ (c). $W(S_1, S_4)$.

Now, a DMP indicates the existence of a region of the *r*th signature which is roughly similar to the corresponding region of the *t*th signature (in the domain d specified by the distance used for DTW). Therefore, for each point of S_r, a score was introduced according to its type of coupling with respect to the points of S_t [29]

$$\mathrm{Score}^t(z_{d,r}^p) = 1 \quad \text{if } z_{d,r}^p \text{ is a DMP, 0 otherwise.} \tag{15}$$

.

The local stability function of S^r was defined as [29]

$$I(z_{d,r}^p) = \frac{1}{N-1} \sum_{\substack{t=1 \\ t \neq r}}^{N} \mathrm{Score}^t(z_{d,r}^p). \tag{16}$$

An example of the computation of the stability index is reported in the following. Let $S = \{S_1, S_2, S_3, S_4\}$ be a set of four (pieces) of signatures of the same writer, Fig. 1 shows the result of the DTW between S_1 and S_2, S_1 and S_3, and S_1 and S_4. Specifically, for S_1 and S_2 it was found that $W*(S_1, S_2) =$ (1,1), (2,2), (2,3), (3,4), (4,5), (5,6), (6,7), (6,8), (7,8), (8,9) [see Fig. 1(a)]. For S_1 and S_3, it was determined that $W*(S_1, S_3) = (1,1), (1,2), (2,3), (3,4), (4,5), (5,5), (5,6), (6,7), (7,8), (8,8)$ [see Fig. 1 (b)]. For S_1 and S_4, it was found that $W*(S_1, S_4) =$ (1, 1), (2, 2), (2, 3), (3, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 8) [see Fig. 1(c)]. The DMP from $W*(S_1, S_2)$ are points $z_{d,1}^1$, $z_{d,1}^3$, $z_{d,1}^4$, $z_{d,1}^5$, and $z_{d,1}^8$. Instead, the DMP from W$*$(S_1, S_3) are points $z_{d,1}^2$, $z_{d,1}^3$, and $z_{d,1}^6$. The DMP from $W*(S_1, S_4)$ are points $z_{d,1}^1$, $z_{d,1}^4$, $z_{d,1}^5$, and $z_{d,1}^6$.

The results are summarized in Table I which also reports the values of the similarity index. As seen in Table I, no region of the signature is very stable (stability index equal to 1). Regions of

TABLE I
LOCAL STABILITY VALUES FOR S_1

	$z^1_{d,1}$	$z^2_{d,1}$	$z^3_{d,1}$	$z^4_{d,1}$	$z^5_{d,1}$	$z^6_{d,1}$	$z^7_{d,1}$	$z^8_{d,1}$
$\text{Score}^2(z^p_{d,r})$	1	0	1	1	1	0	0	1
$\text{Score}^3(z^p_{d,r})$	0	1	1	0	0	1	0	0
$\text{Score}^4(z^p_{d,r})$	1	0	0	1	1	1	0	0
$I(z^p_{d,r})$	**0.66**	**0.33**	**0.66**	**0.66**	**0.66**	**0.66**	**0**	**0.33**

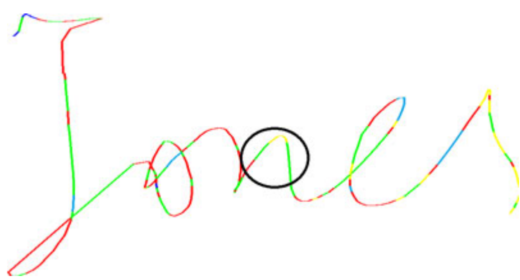

Fig. 2. Example of stability model.

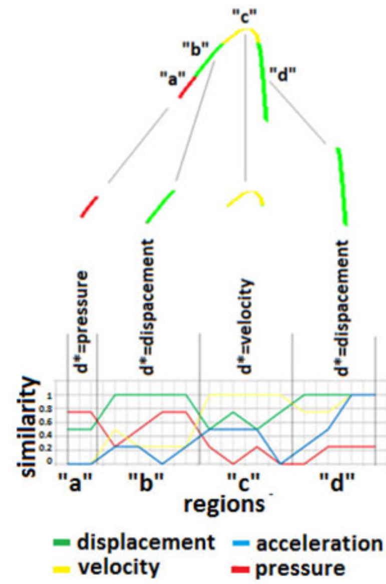

Fig. 3. Segmentation by stability analysis.

medium–high stability (stability index equal to 0.66) are close to points $z^1_{d,1}$, $z^3_{d,1}$, $z^4_{d,1}$, $z^5_{d,1}$, and $z^6_{d,1}$. Regions of medium–low stability (stability index equal to 0.33) are the zone close to point $z^2_{d,1}$, $z^8_{d,1}$; while the zone corresponding to point $z^7_{d,1}$ has very low stability (stability index equal to 0).

According to this strategy, the stability of each part of the prototype signature for each signer in the training phase was estimated in each domain of representation (displacement, velocity, acceleration, and pressure) and used to segment the prototype. The stability model (SM) of the signer was then defined as the way, in which the prototype signature was split into segments

$$SM(S_{n^*}) = ((S_{n^*}(1), d^*(1)), (S_{n^*}(2), d^*(2)), \ldots, (S_{n^*}(v), d^*(v)), \ldots, (S_{n^*}(V), d^*(V))), \tag{17}$$

where $S_{n*}(v)$ is the *v*th segment of the prototype and $d**(v)$ is the most stable domain of representation of $S_{n*}(v)$. In other words, each segment was characterized by the most descriptive specific domain of representation, that is, the representation domain in which the segment of the signature was most stable.

Fig. 2 shows a prototype signature and the stability model, obtained by segmenting the signature according to the most stable domain of representation of each segment: Displacement (green), velocity (yellow), acceleration (blue), and pressure (red).

Fig. 3 shows the process for defining the stability model in more detail. In particular, Fig. 3 shows a specific detail of the signature in Fig. 2 (the part in the circle) for which the analysis of stability was performed and the values of similarity indexes in different domains were graphically reported. This analysis defined the most stable domain of representation $d*$ for each segment of the signature as

1) d* pressure, for segment "a";
2) d* displacement, for segment "b";
3) d* speed, for segment "c";
4) d* displacement, for segment "d".

2) Testing Phase: In the testing phase, each segment of the test signature was classified using the information related to the most stable domain of representation—as specified by the stability model—using a decision tree classifier [40]. In fact, in this paper, we assume that the more a domain is stable for a signature segment, the more difficult it is to imitate it by a forger (although other properties could also be considered). According to the stability model of (17), for each segment $S_{n*}(v)$ of the prototype the C4.5 decision tree classifier [41] was built using the information related to the $d*(v)$ domain of representation of the set of segments (corresponding to $S_{n*}(v)$ of the reference signatures). Both genuine signatures and forgeries are used for reference. The segment $S^{\text{test}}(v)$ of the test signature S^{test} is then verified using the decision tree classifier [in the domain of representation $d*(v)$]. Let $R(v) = 1, if S^{\text{test}}(v)$ is considered to be a genuine segment; and $R(v) = 0$, otherwise. The test signature S^{test} was considered as genuine, if and only if

$$\sum_{\substack{v=1 \\ R(v)=1}}^{V} \text{Length}\,[S_{n^*}(v)] > \sum_{\substack{v=1 \\ R(v)=0}}^{V} \text{Length}[S_{n^*}(v)] \tag{18}$$

where Length $[S_{n*}(v)]$ the length of the segment $S_n*(v)$ (for $v = 1, 2, \ldots, V$), otherwise S^{test} was considered to be forgery. In other words, the test signature was considered genuine if the total length of genuine segments exceeded the total length of forged segments.

III. EXPERIMENTAL RESULTS

Signatures in the SUSIG [42] database of handwritten signatures were used to test the system, according to the leave-one-out strategy. The SUSIG database is composed of two sections: "Blind subcorpus" and "visual subcorpus." This paper used the

TABLE II
SIGNATURE VERIFICATION RESULTS FOR THE SUSIG DATABASE

Approach	FRR (%)	FAR (%)
C. Yuen *et al.* [43]	14.8	2.64
B.Yanikoglu *et al.* [44]	3.03	3.03
K. Wang *et al.* [45]	2.46	2.46
M. I. Khalil *et al.* [46]	3.06	3.06
S. Rashidi *et al.* [47]	2.09	2.09
G. Pirlo *et al.* (this work—using all domains of representation)	3.60	4.15
G. Pirlo *et al.* (this work—using only the most stable domain of representation)	2.15	2.10

"visual subcorpus" which is made up by 100 authors. For each author, 30 signatures were collected (20 genuine and 10 skilled forgeries) [42]: 15 signatures (10 genuine and 5 skilled forgeries) were used for training and 15 (10 genuine and 5 skilled forgeries) for testing, according to a two-fold cross validation strategy.

Table II reports system performance on the SUSIG database. As can be seen in Table II, the verification results of the multidomain system, when each stroke is verified using only the most stable domain of representation, are FRR = 2.15% and FAR = 2.10%, that are comparable with the best results in the literature. It is worth noting that when each stroke is verified using all the domains of representation, the verification results of the same system are FRR = 3.60% and FAR = 4.15%. The studies cited in Table II used a variety of techniques. In [43], the difference between reference and testing signatures was calculated with the standard deviation method and compared with the estimated threshold. To enhance the performance of the system, a probabilistic acceptance model was used. The technique in [44] was based on the fast Fourier transform and used a fixed number of coefficients. The graph representation technique in [45] represented signatures as a series of graphs, whose nodes and edges described some properties at sample points and the relationship between points, respectively. This way, graph matching techniques were used to calculate the distance between graphs. The enhanced DTW-based technique [46] used separated and combined features for signature verification. Curvature change and speed were considered the most efficient features, and pressure was found to give a slight enhancement. The verification process was based on special parameters extracted from a signer reference set. Rashidi *et al.* [47] modeled a velocity signal that was considered stable for the authors. Using pole-zero models based on discrete cosine transform, a precise modeling method was proposed. The signature verification technique used a linear classifier, a parzen window classifier, and a support vector machine. The results obtained demonstrate the validity of the proposed multidomain strategy when compared with many traditional approaches.

IV. CONCLUSION

Stability analysis is useful for a better understanding of the human processes underlying the signing act as well as for assisting in the design of more effective systems for automatic signature verification.

In this paper, a new multidomain system for dynamic signature verification has been presented. The approach assumes that for each part of a handwritten signature a domain of representation (e.g., position, velocity, acceleration, or pressure) exists in which a signer is more stable. Furthermore, the method exploits the notion that the domain of representation in which the signer is more stable will be more difficult for a forger to imitate than other representation domains in the signature. Indeed, even small variations from the typical behavior of a signer can be easily detected in a high stability domain. Hence, the technique proposed here uses a stability model that first describes the most stable domain of representation for each part of a signature. Then, in the verification process, signature matching can be performed using only the most stable domain of representation in each part of the signature. The experimental results were carried out using the SUSIG public database. The results demonstrated the effectiveness of the proposed technique when compared with other approaches in the literature.

In conclusion, this paper shows that stability analysis can be used to adapt verification parameters to a signer's specific characteristics; thus, offering useful information for the design of more rational and effective signature verification techniques. It is worth noting that this research also offers new insights for further investigation, like for instance, those related to the optimal distance for the stability analysis, to the analysis of the effect of using multiple prototypes for characterizing the human stability in signing, and to the analysis of the most profitable technique for estimating the local stability in handwritten signatures. Finally, the role of stability has been addressed in this paper only considering the most stable domain of representation of each region of a signature. In addition, domains with high variability can probably provide very distinctive characteristics that could be considered. In a similar way, both stable and variable regions of a signature could be considered for supporting the advanced personalized approaches in signature verification, since probably, from a behavioral point of view, a variability model of a signer could also be very informative and complementary to a stability model.

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